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DIRECT AND INDIRECT REQUIREMENTS FOR GROSS OUTPUT IN INPUT-OUTPUT ANALYSIS (*)

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I.

In input-output analysis the concept of total (direct and indirect) requirements is usually related to sectoral final outputs. For each particular final product Y_i total output requirements and total primary input requirements are measured respectively by:

$$X_Y^{(i)} = (I-A)^{-1} Y^{(i)} \tag{1a}$$

and

$$W_Y^{(i)} = A_w (I-A)^{-1} Y^{(i)} \tag{1b}$$

where

$X_Y^{(i)}$ = vector of sectoral gross outputs required directly and indirectly by $Y^{(i)}$;

$W_Y^{(i)}$ = vector of primary inputs required directly and indirectly by $Y^{(i)}$;

A = intermediate input coefficient matrix;

A_w = primary input coefficient matrix;

$Y^{(i)}$ = vector of elements equal to zero except the i-th one, which is equal to Y_i.

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The total requirements $X_Y^{(i)}$ and $W_Y^{(i)}$ appeared for the first time in the well-known empirical investigations of Leontief ([1]) and are also indicated as « sub-systems » ([2]) or as « vertically integrated sectors » ([3]). They permit us to split the economic system into ideal self-sufficient sectors which produce final commodities and all their necessary intermediate inputs. The i-th column of the Leontief inverse $(I-A)^{-1}$ contains the quantities of commodities that are directly and indirectly required to obtain one unit of the i-th final output ([4]).

In this paper we present a measure of total requirements relative to *gross* outputs rather than *final* outputs. To our knowledge this concept has never been explicitly developed in input-output analysis although it can be related to the global effects on the economy stemming from the existence of certain domestic industries. In particular it may be interesting to compute total requirements of an industry even in the case where its production does not include any final commodities ([5]).

II.

Let us divide the n industries of the input-output table into two groups: the group 1 consisting of the first l industries (where $1<l<n$) and the group 2 consisting of the last $m=(n-l)$ industries. The input coefficient matrices can then be partitioned as follows:

$$A=\left[\begin{array}{c:c} A_{11} & A_{12} \\ \hdashline A_{21} & A_{22} \end{array}\right] \quad \text{and} \quad A_w=[A_{w1} \,\vdots\, A_{w2}]$$

where

A_{ij} = intermediate input coefficient matrix relative to group j and to inputs supplied by group i;

A_{wj} = primary input coefficient matrix relative to group j;

([1]) See, for example, Leontief (1941), (1953), (1956).
([2]) See Sraffa (1960, p. 89).
([3]) See, among others, Pasinetti (1973).
([4]) Parikh (1975) gives a survey of various alternative specifications of this concept.
([5]) In this case the computation of total requirements for final output is of course ruled out since in the traditional Leontief's formulae (1*a*) and (1*b*) $Y_i=0$ and therefore $X_Y^{(i)}=0$ and $W_Y^{(i)}=0$.

and we have the following partitioned input-output system in reduced form:

$$\begin{bmatrix} X_1 \\ \hdashline X_2 \end{bmatrix} = \begin{bmatrix} (I_l - A_{11}) & -A_{12} \\ \hdashline -A_{21} & (I_m - A_{22}) \end{bmatrix}^{-1} \cdot \begin{bmatrix} Y_1 \\ \hdashline Y_2 \end{bmatrix} \tag{2a}$$

and

$$W = [A_{w1} \vdots A_{w2}] \begin{bmatrix} X_1 \\ \hdashline X_2 \end{bmatrix} \tag{2b}$$

where

Y_i = vector of final outputs of group i;

X_j = vector of total outputs of group j required by Y_1 and Y_2;

W = vector of primary inputs required by Y_1 and Y_2.

Applying Frobenius-Schur's formulae for the inversion of partitioned matrices, the system $(2a)$ can be rewritten as follows:

$$\begin{bmatrix} X_1 \\ \hdashline X_2 \end{bmatrix} = \begin{bmatrix} N & NT_1 \\ \hdashline T_2N & T + T_2NT_1 \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ \hdashline Y_2 \end{bmatrix} \tag{3a}$$

or, in other terms,

$$\begin{bmatrix} X_1 \\ \hdashline X_2 \end{bmatrix} = \begin{bmatrix} B + B_2MB_1 & B_2M \\ \hdashline MB_1 & M \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ \hdashline Y_2 \end{bmatrix} \tag{3b}$$

where

$B = (I_l - A_{11})^{-1}$; $T = (I_m - A_{22})^{-1}$; $B_1 = A_{21}B$; $B_2 = BA_{12}$;
$T_1 = A_{12}T$; $T_2 = TA_{21}$; $N = LB$; $L = (I_l - B_2T_2)^{-1}$;
$M = KT$; $K = (I_m - T_2B_2)^{-1}$.

The matrix

$$\begin{bmatrix} N \\ \hdashline T_2N \end{bmatrix} = \begin{bmatrix} B + B_2MB_1 \\ \hdashline MB_1 \end{bmatrix}$$

is the traditional total output-requirement matrix for one unit of the first l final commodities, while the matrix

$$\begin{bmatrix} NT_1 \\ \hdashline T + T_2 NT_1 \end{bmatrix} = \begin{bmatrix} B_2 M \\ \hdashline M \end{bmatrix}$$

is the traditional total output-requirement matrix for one unit of the last $(n-l)$ final commodities. Moreover B and T are what Miyazawa (1966) called « internal matrix multipliers » relative only to internal propagation activities inside the 1st and the 2nd groups of industries respectively, while L and K are what Miyazawa (1966) called « external matrix multipliers » relative to external propagation activities between the 1st and the 2nd groups of industries. $T_2NT_1 = (I_m - A_{22})^{-1}A_{21}NA_{12}(I_m - A_{22})^{-1}$ and $B_2MB_1 = (I_l - A_{11})^{-1}A_{12}MA_{21}(I_l - A_{11})^{-1}$ are induced sub-matrix multipliers which measure the production of one group of industries required indirectly by the same group of industries through the inputs provided directly and indirectly by the other group of industries.

The solutions (3a) and (3b) are economically significant if and only if the Hawkins-Simon (H-S) conditions are satisfied, that is the vector $[X_1 \vdots X_2]'$ is non-negative for any set of non-negative vectors $[Y_1 \vdots Y_2]'$ (and the Leontief inverse $(I_n - A)^{-1}$ is a non-negative matrix) if and only if the upper left-hand principal minors of the matrix $(I_n - A)$ are positive. As often recognized, the meaning of these conditions is not immediate (6). However the partitioned form of the input-output system permits a clear reading of their economic implications. Consider the first group as formed by only one industry ($l = 1$) and the second group as formed by the remaining $(n-1)$ industries. Then we have

$$(I - A) = \begin{bmatrix} 1 - a_{11} & \vdots & -a'_{12} \\ \hdashline -a_{21} & \vdots & I_{n-1} - A_{22} \end{bmatrix}$$

where the matrix symbols are self-explainatory and of proper order.

In (3a), from the definition of the inverse matrix,

$$N = \frac{|I_{n-1} - A_{22}|}{|I_n - A|}.$$

(6) On this subject Woods (1978), for example, claims: « conditions involving the signs of principal minors are notoriously difficult to explain » (p. 4).

It can be shown that (7)

$$N^{-1} = \frac{|I_n - A|}{|I_{n-1} - A_{22}|} = 1 - a_{11} - a'_{12}(I_{n-1} - A_{22})^{-1} a_{21} .$$

If the H-S conditions are satisfied,

$$|I_{n-1} - A_{22}| > 0$$

and

$$|I_n - A| > 0 .$$

Therefore

$$1 > a_{11} + a'_{12}(I_{n-1} - A_{22})^{-1} a_{21}$$

which means that the unit of gross output of a commodity must use less than one unit of itself as direct (a_{11}) and indirect $(a'_{12}(I_{n-1} - A_{22})^{-1} a_{21})$ input (8). This garantees, for every $X_1 > 0$, the positivity of net output given by $[1 - a_{11} - a'_{12}(I_{n-1} - A_{22})^{-1} a_{21}]X_1$. From (3*a*) it is evident that

$$[1 - a_{11} - a'_{12}(I_{n-1} - A_{22})^{-1} a_{21}]X_1 = F_1$$

where $F_1 = Y_1 + T_1 Y_2$. In other terms, the net output has to satisfy what can be defined as « total external demand » of the industry, i.e. the own final demand (Y_1) plus the demand of the other industries $(T_1 Y_2 = a'_{12} \cdot \cdot (I_{n-1} - A_{22})^{-1} Y_2)$.

Returning to the general case in which $1 < l < n$ and $m = (n - l)$, in (3*a*) and (3*b*) total gross outputs are given by $X_1 = [N \vdots NT_1][Y_1 \vdots Y_2]'$ and $X_2 = [MB_1 \vdots M][Y_1 \vdots Y_2]'$. Therefore total gross outputs can be divided into two parts: the direct and indirect output requirements (NY_1 and MY_2) for the own final demand and the direct and indirect output

(7) See the matrix definitions given for (3*a*).

(8) The economic interpretation of the H-S conditions in production activities should in fact be referred, for each industry, to direct and indirect requirements of its own product per unit of *gross* output (not per unit of final or net output). (See, for example, Baumol, 1979 and Jeong, 1982 for a correct economic interpretation of the H-S conditions).

requirements (NT_1Y_2 and MB_1Y_1) for final demand of the other industries. Moreover, the total output of one group of industries induced directly and indirectly by the production activities of the other group is given by:

$$X_2^{(1)} = T_2X_1 = [T_2N \,\vdots\, T_2NT_1]\begin{bmatrix} Y_1 \\ \cdots \\ Y_2 \end{bmatrix} \tag{4a}$$

and

$$X_1^{(2)} = B_2X_2 = [B_2MB_1 \,\vdots\, B_2M]\begin{bmatrix} Y_1 \\ \cdots \\ Y_2 \end{bmatrix} \tag{4b}$$

where $T_2 = (I_m - A_{22})^{-1}A_{21}$ is the total requirement matrix for the output of the last $(n-l)$ industries relative to one unit of gross output of the first l industries, and $B_2 = (I_l - A_{11})^{-1}A_{12}$ is the total requirement matrix for the output of the first industries relative to one unit of gross output of the last $(n-l)$ industries.

Taking account of (4*a*) and (4*b*) total output requirements for gross production of one industry or of one group of industries can be computed simply by eliminating the internal matrix multipliers of the other group of industries from the Leontief inverse [9]:

$$\begin{bmatrix} X_1 \\ \cdots \\ X_2^{(1)} \end{bmatrix} = \left\{\begin{bmatrix} N & NT_1 \\ T_2N & T+T_2NT_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & T \end{bmatrix}\right\} \cdot \begin{bmatrix} Y_1 \\ \cdots \\ Y_2 \end{bmatrix}, \tag{5a}$$

$$\begin{bmatrix} X_1^{(2)} \\ \cdots \\ X_2 \end{bmatrix} = \left\{\begin{bmatrix} B+B_2MB_1 & B_2M \\ MB_1 & M \end{bmatrix} - \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}\right\} \cdot \begin{bmatrix} Y_1 \\ \cdots \\ Y_2 \end{bmatrix}, \tag{5b}$$

where $\begin{bmatrix} X_i \\ \cdots \\ X_j^{(i)} \end{bmatrix}$ = vector of total output requirements for gross output of industry i.

Total intermediate input requirements for gross outputs X_1 and X_2 are, therefore, obtained by subtracting the net output from the total

(9) This method corresponds to the hypothetical extraction method set up by Strassert (1968) and Schultz (1976), (1977) for the identification of linkages of key sectors in the economy. They removed the column and the row relative to a particular industry from the Leontief matrix and subtracted the corresponding reduced inverse matrix from the original one.

output requirements:

$$\begin{bmatrix} X_1^{(1)} \\ \cdots \\ X_2^{(1)} \end{bmatrix} = \begin{bmatrix} X_1 \\ \cdots \\ X_2^{(1)} \end{bmatrix} - \begin{bmatrix} Y_1 + T_1 Y_2 \\ \cdots \\ 0 \end{bmatrix}, \tag{6a}$$

$$\begin{bmatrix} X_1^{(2)} \\ \cdots \\ X_2^{(2)} \end{bmatrix} = \begin{bmatrix} X_1^{(2)} \\ \cdots \\ X_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \cdots \\ B_1 Y_1 + Y_2 \end{bmatrix}, \tag{6b}$$

where $(Y_1 + T_1 Y_2)$ and $(B_1 Y_1 + Y_2)$ are equal to the net outputs of the first and the second group of industries respectively, and $X_1^{(1)}$ and $X_2^{(2)}$ are the direct and indirect requirements of the first and the second group of industries for their own products.

Total primary factor requirements of gross outputs are computed by premultiplying (5*a*) and (5*b*) by the matrices of primary input coefficients A_{w1} and A_{w2}:

$$W^{(1)} = [A_{w1} \vdots A_{w2}] \begin{bmatrix} X_1 \\ \cdots \\ X_2^{(1)} \end{bmatrix}, \tag{7a}$$

$$W^{(2)} = [A_{w1} \vdots A_{w2}] \begin{bmatrix} X_1^{(2)} \\ \cdots \\ X_2 \end{bmatrix}, \tag{7b}$$

which can be subdivided into direct factor requirements ($A_{w1} X_1$ and $A_{w2} X_2$) and indirect factor requirements ($W^{(1)} - A_{w1} X_1 = A_{w2} X_2^{(1)}$ and $W^{(2)} - A_{w2} \cdot X_2 = A_{w1} X_1^{(2)}$).

While in (1*a*)-(1*b*) $\sum_i X_Y^{(i)} = X$ and $\sum_i W_Y^{(i)} = W$, the sum of total output and primary input requirements for gross output, which are given by (5*a*)-(5*b*) and (7*a*)-(7*b*), leads to a double counting procedure because some direct requirements of one industry are indirect requirements of other industries which use part of the output of that industry as an intermediate input. Nevertheless the concept of total requirements for gross output may be helpful in the analysis of the influence of one single industry or one group of industries on the rest of the economy.

III.

The method described above has been applied to the Italian input-output table for 1975 in order to estimate total employment requirements

TAB 1 - DIRECT AND INDIRECT EMPLOYMENT REQUIREMENTS OF INDUSTRIAL BRANCHES OF THE ITALIAN ECONOMY, YEAR 1975 (WORKERS)

BRANCHES	NACE -CLIO(*) CLASSIF. CODE	TOTAL EMPLOYM. REQUIREM. OF FINAL OUTPUT (A)	TOTAL EMPLOYM. REQUIREM. OF GROSS OUTPUT (B=C+D)	(A) --- (B) %	DIRECT EMPLOYM. REQUIREM. OF GROSS OUTPUT (C)	(C) --- (B) %	INDIRECT EMPLOYM. REQUIREM. OF GROSS OUTPUT (D)	(D) --- (B) %
1 AGRICULTURAL, FORESTRY AND FISHERY PRODUCTS	(01)	1574840	3269443	48.2	3047000	93.2	222443	6.8
2 COAL, LIGNITE (BROWN COAL) AND BRIQUETTES	(03)	0	1132	0.0	802	70.9	330	29.1
3 PRODUCTS OF COKING	(05)	2793	12628	22.1	2098	16.6	10530	83.4
4 CRUDE PETROLEUM, NATURAL GAS AND PETROLEUM PRODUCTS	(07)	35985	72523	49.6	27450	37.9	45073	62.1
5 ELECTRIC POWER, GAS, STEAM AND WATER	(09)	33962	216976	15.7	149520	68.9	67456	31.1
6 PRODUCTION AND PROCESSING OF RADIOACTIVE MATERIALS & ORES	(11)	0	0	--	0	--	0	--
7 FERROUS AND NON-FERROUS ORES AND METALS	(13)	95678	484679	19.7	274111	56.6	210568	43.4
8 NON-METAL MINERAL PRODUCTS	(15)	126422	557598	22.7	413877	74.2	143721	25.8
9 CHEMICAL AND PHARMACEUTICAL PRODUCTS	(17)	319261	602983	52.9	307855	51.1	295128	48.9
10 METAL PRODUCTS, EXCEPT MACHINERY AND TRANSPORT EQUIPMENT	(19)	280576	645770	43.4	417254	64.6	228516	35.4
11 AGRICULTURAL AND INDUSTRIAL MACHINERY	(21)	669670	757602	88.4	402916	53.2	354686	46.8
12 OFFICE AND DATA PROCESSING MACHINES, PRECISION INSTRUM.	(23)	112176	140325	79.9	88715	63.2	51610	36.8
13 ELECTRICAL GOODS	(25)	446419	604257	73.9	394949	65.4	209308	34.6
14 MOTOR VEHICLES	(27)	425059	487414	87.2	242629	49.8	244785	50.2
15 OTHER TRANSPORT EQUIPMENT	(29)	177902	220687	80.6	131025	59.4	89662	40.6
16 MEAT, MEAT PREPARATIONS AND PRESERVES	(31)	558181	661665	84.4	66758	10.1	594907	89.9
17 MILK AND DAIRY PRODUCTS	(33)	298724	350393	85.3	45055	12.9	305338	87.1
18 OTHER FOOD PRODUCTS	(35)	875632	1130283	77.5	279402	24.7	850881	75.3
19 BEVERAGES	(37)	80226	130794	61.3	49596	37.9	81198	62.1
20 TOBACCO PRODUCTS	(39)	35919	35941	99.9	13371	37.2	22570	62.8
21 TEXTILES AND CLOTHING	(41)	1295807	1404605	92.3	1185716	84.4	218889	15.6
22 LEATHERS, LEATHER AND SKIN GOODS, FOOTWEAR	(43)	317299	337879	93.9	244724	72.4	93155	27.6
23 TIMBER, WOODEN PRODUCTS AND FURNITURE	(45)	402900	614154	65.6	495526	80.7	118628	19.3
24 PAPER AND PRINTING PRODUCTS	(47)	127365	378447	33.7	260435	68.8	118012	31.2
25 RUBBER AND PLASTICS	(49)	125990	340870	37.0	214709	63.0	126161	37.0
26 OTHER MANUFACTURES	(51)	119997	125770	95.4	82320	65.5	43450	34.5
27 BUILDING AND CONSTRUCTION	(53)	2228741	2565129	86.9	1749184	68.2	815945	31.8
28 RECOVERY AND REPAIR SERVICES	(55)	283659	678741	41.8	506533	74.6	172208	25.4
29 WHOLESALE AND RETAIL TRADE	(57)	2318635	2842722	81.6	2388957	84.0	453765	16.0
30 LODGING AND CATERING SERVICES	(59)	956574	1068625	89.5	585808	54.8	482817	45.2
31 INLAND TRANSPORTS	(61)	446811	855683	52.2	702994	82.2	152689	17.8
32 MARITIME AND AIR TRANSPORTS	(63)	110145	122160	90.2	64125	52.5	58035	47.5
33 AUXILIARY TRANSPORT SERVICES	(65)	49438	207395	23.8	140370	67.7	67025	32.3
34 COMMUNICATIONS	(67)	101426	282345	35.9	226273	80.1	56072	19.9
35 SERVICES OF CREDIT AND INSURANCE INSTITUTIONS	(69)	25721	401075	6.4	265348	66.2	135727	33.8
36 BUSINESS SERVICES PROVIDED TO ENTERPRISES	(71)	119942	512865	23.4	435216	84.9	77649	15.1
37 SERVICES OF RENTING OF IMMOVABLE GOODS	(73)	187154	232887	80.4	0	0.0	232897	100.0
38 MARKET SERVICES OF EDUCATION AND RESEARCH	(75)	80682	138015	58.5	126760	91.8	11255	8.2
39 MARKET SERVICES OF HEALTH	(77)	167692	198832	84.3	169513	85.3	29319	14.7
40 RECREATIONAL AND CULTURAL AND OTHER MARKET SERVICES	(79)	463091	546834	84.7	464470	84.9	82364	15.1
41 GENERAL PUBLIC SERVICES	(81)	1849669	1851796	99.8	1503824	81.2	347972	18.8
42 NON-MARKET SERVICES OF EDUCATION AND RESEARCH	(85)	815837	822795	99.2	759676	92.3	63119	7.7
43 NON-MARKET SERVICES OF HEALTH	(89)	547543	549367	99.7	392500	71.4	156867	28.6
44 DOMESTIC SERVICES AND OTHER NON-MARKET SERVICES N.E.C	(93)	535142	535142	100.0	506400	94.6	28742	5.4
TOTAL	(99)	19825764	(**)	--	19825764	--	(**)	--

(*) GENERAL INDUSTRIAL CLASSIFICATION OF ECONOMIC ACTIVITIES WITHIN THE EUROPEAN COMMUNITIES - INPUT-OUTPUT CLASSIFICATION

(**) THIS TOTAL IS NOT GIVEN BECAUSE IT CONTAINS DOUBLE-COUNTED ELEMENTS of final and gross outputs of each industry [10]. The computations were made for one industry at a time by applying (*1b*) and (*7b*) in which $l = 1$ and $m = (n - 1)$. The results are shown in the first and second columns of Table 1 where the two concepts of total labor requirements can be empirically compared. For further information, the fourth and sixth columns of Table 1 contain the breakdown of total labor requirements of gross outputs into direct and indirect labor requirements. As might be expected, the higher differences between the total labor requirements of gross and final outputs are found in basic industries which sell most of their outputs as intermediate inputs to other industries, while small differences are found in industries which mainly produce for final demand. The first group of industries include « Products of coking », « Electric power, gas, steam and water », « Ferrous and non-ferrous ores and metals », « Non-metallic mineral products », where total labor requirements of final output is less than 30 per cent of those of gross output. The second group of industries include « Motor vehicles », « Meat, meat preparations and preserves », « Other food products », « Textiles and clothing », « Leather and skin products, footwear » with labor requirements of final output very close to those of gross output. In conclusion, these empirical findings confirm that the direct and indirect effects on production and on factor employment originating from the existence of particular domestic industries cannot be exhaustively estimated by applying the traditional concept of total requirements for final output. They imply that many studies on interindustry relations, such as those on the interdependence between service and goods-producing sectors, might conveniently be carried out using the concept of direct and indirect requirements for gross output.

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(10) This table and the sectoral employment data have been published by Italy's Central Statistical Office (see ISTAT, 1981).

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